# Bond Futures

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## BOND FUTURES CONTRACTS

### A. Introduction and Overview of Contracts

There are many different futures contracts today covering changes in the value of government bonds. The oldest and most prominent in terms of liquidity refer to U.S. Treasury bonds. But government bond futures now exist for nearly every liquidly traded government bond market. For a complete list, please see the appendix attached at the end of this guide.

Features which all government bond futures contracts have in common include contracts for the longest maturities of actively issued government bonds, specific deliverable bonds, and tick values equal to the smallest price change of the underlying bonds themselves.

Government bond futures contracts are used by portfolio managers, bond traders, long-term interest rate speculators, swap book managers, and bond issuers and underwriters. Government bond futures are also used to hedge the risk of eurobonds, corporate bonds, and other forms of long-term debt, to varying degrees of success. The fact is that there is often little correlation among government bond rates and other bond rates in the same currency over short periods of time, like a trading day or two. Over longer periods of time, long-term rates in the same currency seem to move more or less in tandem. Over shorter periods, however, the spreads separating them can change dramatically, destroying well-calculated hedge ratios and turning *hedges* into additional *positions*.

**Users of Long Term Interest Rate Futures**

Managing Pension Funds

One of the main users of long term interest rate futures contracts are pension fund managers. These managers are responsible for creating an asset portfolio whose value coincides with or exceeds the value of a given schedule of liabilities (anticipated payouts to retirees), and whose duration or interest rate sensitivity is the same as the liabilities. Fund managers refer to this as *immunizing* their liabilities against a certain range of interest rate movements.

Locking In Cost Of Debt Issuance

Very often corporations know in advance that they will be issuing debt at some point in the near future. One example would be future project funding. The corporation could issue the debt today and invest the proceeds until such time that the funds are actually required. By using the interest rate futures contract the risk free component of the issuance can be locked in.

The corporation would sell futures contracts. If interest rates rose between now and the time of debt issuance, the cost of the debt would increase. However this would be offset

by a gain on the futures transaction. When the company closes its futures position the price will have fallen due to an increase in yields. This gain could be used to offset the increased expense on the debt.

Locking In Value Of An Asset

If a firm has an asset (bond) that they know they will want to sell in the future and the view is that interest rates are going to increase between now and the time of anticipated sale, by using the futures market the value of the asset at sale can be locked in. The bond could be sold today and the proceeds reinvested but this may not be feasible. An upcoming coupon payment may be desirable. The firm may want to wait for rates to rise before investing again.

In this case a long-term interest rate future would be sold. If rates increased the value of the asset would decline. This would be offset by a gain in the futures contracts. As rates increased the price of the futures would fall and the contract could be closed out at a gain.

Bond Traders

The long term interest rate futures add another tool to the bond trader's portfolio. They can be used to hedge an existing portfolio or adjust the duration of the existing portfolio. Traders can adjust their view and the portfolio's position quickly by using the futures market.

## B. Contract Specifications

This guide will use two well-known futures contracts to explore how bond futures work, Treasury bond futures and *Bund* futures. While there are many others which are just as useful and just as interesting, the terms and functioning of bond futures contracts are fairly homogenous from exchange to exchange, so the two selected will prove representative around the world.

Bonds issued by the government of the United States are called United States Treasury bonds, and are often referred to simply as "Treasuries." Bonds issued by the government of Germany are called "*Anleihe der Bundesrepublik Deutschland*", referred to as *Bunds*, "*Anleihe der Bundesrepublik Deutschland — Fonds Deutsche Einheit*", referred to as German unity bonds, and "*Anleihe der Treuhandanstalt*", referred to as *Treuhand* bonds. All three types of German government bonds are included in *Bund* futures. *Bundesrepublik* means the Federal Republic, so calling German government bonds *Bunds*, means we are calling them *Federals*.



U.S. Treasury bond futures contracts are traded on the Chicago Board of Trade (CBOT) and several other exchanges, including the London International Financial Futures and Options Exchange (LIFFE). *Bund* futures contracts are traded on LIFFE and on the Deutsche Termin Börse (DTB). Specifications for these bond futures contracts follow.

| | ***Treasury Bond*** | ***10-Year Treasury Note*** | ***Bund*** |
|---|---|---|---|
| Trading Unit | One U.S. Treasury bond having a face value at maturity of $100,000 or multiple thereof | One U.S. Treasury note having a face value at maturity of $100,000 or multiple thereof | DM250,000 nominal value German Government Bond (*Bund*) |
| Notional Tenor | 30 years | 10 years | 10 years |
| Theoretical Coupon: | 8% | 8% | 6% |
| Deliverable Bonds | U.S. Treasury bonds that have a maturity (or a first callable date) of at least 15 years from the first business day of the delivery month. | U.S. Treasury notes that have a maturity of at least 6½ years, but not more than 10 years, from the first business day of the delivery month. | German government bonds (as defined above) with 8½-10 years remaining maturity as at the 10th calendar day of the delivery month. |
| Price Quote | Points ($1,000) and thirty-seconds of a point | Points ($1,000) and thirty-seconds of a point | Percent of nominal value to two decimal points |
| Tick Size | 1/32 of a point ($31.25) | 1/32 of a point ($31.25) | 0.01 (DM25) |
| Daily Price Limit | 3 points ($3,000); lifted the 2nd business day before the 1st day of the delivery month. | 3 points ($3,000); lifted the 2nd business day before the 1st day of the delivery month. | None |
| Contract Months | Mar, Jun, Sep, Dec | Mar, Jun, Sep, Dec | Mar, Jun, Sep, Dec |
| Delivery Day | Last business day of contract month | Last business day of contract month | 10th calendar day of contract month |
| Last Trading Day | 7th Chicago business day before delivery day | 7th Chicago business day before delivery day | 3 Frankfurt working days prior to delivery day |

## C. Bond Futures Prices

### Deliverable Bonds

The list of bonds which fits the description given above for each contract changes from time to time as new bonds are issued and as older bonds become shorter than the minimum maturity described above.

In general terms, bonds which are cheaper to buy are also the cheaper bonds to deliver against the futures contract. The cheapest bond to deliver also changes from time to time, normally as a function of changing bond yields or a change in the slope of the yield curve.

Bonds which are deliverable but not generally desired by bond investors tend to move up the list of cheapest to deliver bonds because their prices are discounted in the cash bond

market. Under current market conditions (long-term interest rates at historical cyclical lows) this is true for bonds issued earlier, which have very high coupons. In a few years, when long-term yields have risen again, bonds being issued now will be available at a discount in the cash market, and so will likely remain somewhat more desirable.

When bond yields are low, as they are in late 1993, bonds with shorter modified duration (henceforth referred to simply as *duration*) tend to be cheaper to deliver. This is because market prices for shorter duration bonds rise relatively less than market prices for longer duration bonds as rates fall. Higher coupons shorten duration. Thus, the cheapest to deliver bonds when yields in the market are low will be those bonds with the highest coupons. This is clearly evident with the deliverable bonds into the long Treasury bond contract on the CBOT.

As of November 1993, the list of deliverable bonds for the March 1994 Treasury bond futures contract stood as follows:

**Deliverable Bonds, CBOT Treasury Bond Futures, March 1994**

| Bond | Coupon | Maturity | Market Price | Yield | Modified Duration | Conversion Factor |
|---|---|---|---|---|---|---|
| Treasury | 11 3/4 | 15/11/2009 | 156 19/32 | 6.1436% | 8.8916 | 1.3298 |
| Treasury | 13 1/4 | 15/5/2009 | 171 13/32 | 6.0737% | 8.5605 | 1.4539 |
| Treasury | 12 1/2 | 15/8/2009 | 164 3/32 | 6.1050% | 8.5638 | 1.3921 |
| Treasury | 11 1/4 | 15/2/2015 | 156 13/32 | 6.3680% | 10.2078 | 1.3262 |
| Treasury | 10 5/8 | 15/8/2015 | 149 1/2 | 6.3823% | 10.4223 | 1.2659 |
| Treasury | 9 7/8 | 15/11/2015 | 140 25/32 | 6.3956% | 10.7908 | 1.1910 |
| Treasury | 9 1/4 | 15/2/2016 | 133 13/32 | 6.4097% | 10.7838 | 1.1277 |
| Treasury | 8 3/4 | 15/5/2017 | 127 7/8 | 6.4335% | 11.3486 | 1.0783 |
| Treasury | 8 7/8 | 15/8/2017 | 129 7/16 | 6.4375% | 11.1842 | 1.0915 |
| Treasury | 9 1/8 | 15/5/2018 | 132 29/32 | 6.4374% | 11.4776 | 1.1192 |
| Treasury | 7 1/2 | 15/11/2016 | 112 5/8 | 6.4402% | 11.5586 | 0.9482 |
| Treasury | 7 1/4 | 15/5/2016 | 109 21/32 | 6.4320% | 11.5242 | 0.9229 |
| Treasury | 8 7/8 | 15/2/2019 | 130 1/16 | 6.4465% | 11.4873 | 1.0935 |
| Treasury | 9 | 15/11/2018 | 131 23/32 | 6.4322% | 11.6118 | 1.1067 |
| Treasury | 8 1/8 | 15/8/2019 | 120 23/32 | 6.4617% | 11.7525 | 1.0133 |
| Treasury | 8 1/2 | 15/2/2020 | 125 21/32 | 6.4569% | 11.7578 | 1.0540 |
| Treasury | 8 3/4 | 15/5/2020 | 129 | 6.4517% | 11.9462 | 1.0816 |
| Treasury | 8 3/4 | 15/8/2020 | 129 1/16 | 6.4533% | 11.7952 | 1.0816 |
| Treasury | 8 1/8 | 15/5/2021 | 121 11/32 | 6.4562% | 12.2737 | 1.0137 |
| Treasury | 7 7/8 | 15/2/2021 | 118 | 6.4610% | 12.0927 | 0.9861 |
| Treasury | 8 1/8 | 15/8/2021 | 121 3/8 | 6.4578% | 12.1169 | 1.0136 |
| Treasury | 8 | 15/11/2021 | 120 1/32 | 6.4457% | 12.4015 | 1.0000 |
| Treasury | 7 5/8 | 15/11/2022 | 115 17/32 | 6.4360% | 12.6771 | 0.9581 |
| Treasury | 7 1/4 | 15/8/2022 | 110 15/32 | 6.4443% | 12.5363 | 0.9163 |
| Treasury | 7 1/8 | 15/2/2023 | 109 15/32 | 6.4036% | 12.6922 | 0.9019 |
| Treasury | 6 1/4 | 15/8/2023 | 101 5/32 | 6.1638% | 13.3184 | 0.8032 |

The bonds above are ranked in order of relative price, with the cheapest to deliver bond at the top of the list. Duration ranges from only 8.89 years for the cheapest to deliver to 13.3 years for the most recent long bond issued in August 1993. Duration appears to be the principal determinant of which bond is cheaper to deliver, due to the very low level of yields in the market. In addition, bonds with very high coupons (the first three bonds on the list were issued at the top of the interest rate cycle in 1984) trade at very high premiums above par. Many investors are loathe to spend 150% of par or more to buy a bond.

When bond yields are relatively high (above 8%), the reverse is true. Bonds with relatively longer duration will tend to be the cheaper bonds to deliver. The market prices of longer duration bonds tend to fall relatively faster than the market prices of shorter duration bonds as yields rise. They are thus relatively cheaper to buy and therefore cheaper to deliver.

Duration is not the only factor in determining which bonds are cheaper to deliver. Bonds' prices relative to each other are affected by other variables such as liquidity, whether a bond is callable, and perceived differential credit quality among eligible "government" issuers. In general, when bonds have the same duration, the bond with the higher yield will have the lower price, and therefore be cheaper to deliver. Its price is lower because it is less desirable to investors for one of the reasons listed above.

The list of deliverable bonds is much shorter for *Bund* futures, as the German government has in the past not needed to fund such large deficits as the U.S. government. Reunification has conspired to add a great deal of "liquidity" to the German government bond market however, and the list of deliverable bonds grows longer.

Coupon size and duration are a factor in the cheapest to deliver against *Bund* futures, too, but there is another factor of even greater importance. *Treuhand* bonds, issued by a German government guaranteed agency set up to fund the purchase and privatization of companies in the former East Germany, enjoy the full legal backing of the Federal Republic of Germany, and are thus deliverable into *Bund* futures contracts on LIFFE and DTB. But they are not held in equal esteem in the cash bond market, and normally trade at prices discounted slightly from *Bunds* so as to yield 10-15 basis points more. Since their prices are lower, they become cheaper to deliver almost automatically. The effects of duration and liquidity are the major differentiation among the seven *Treuhand* issues currently deliverable. *THA* stands for *Treuhand* bonds, and *DBR* stands for *Bunds*.

### LIFFE *Bund* Futures, December 1993

| Bond | Coupon | Maturity | Price | Yield | Modified Duration | Conversion Factor |
|---|---|---|---|---|---|---|
| THA | 7 1/8 | 29/1/2003 | 107.363 | 6.0521% | 6.3839 | 1.076897 |
| THA | 6 7/8 | 11/6/2003 | 105.894 | 6.0381% | 6.7737 | 1.061493 |
| THA | 7 3/8 | 2/12/2002 | 109.050 | 6.0441% | 6.1953 | 1.092700 |
| THA | 7 3/4 | 1/10/2002 | 111.613 | 6.0166% | 6.4053 | 1.116075 |
| THA | 6 1/2 | 23/4/2003 | 103.425 | 6.0068% | 6.7183 | 1.034535 |
| THA | 6 5/8 | 9/7/2003 | 104.363 | 6.0089% | 6.8949 | 1.043798 |
| DBR | 7 1/8 | 20/12/2002 | 107.956 | 5.9626% | 6.2951 | 1.076519 |
| THA | 6 | 12/11/2003 | 100.238 | 5.9670% | 7.3390 | 0.999868 |
| DBR | 7 1/4 | 21/10/2002 | 108.800 | 5.9472% | 6.5354 | 1.083526 |
| DBR | 6 3/4 | 22/4/2003 | 105.750 | 5.9290% | 6.6834 | 1.051997 |
| DBR | 8 | 22/7/2002 | 113.706 | 5.9248% | 6.2036 | 1.130619 |
| DBR | 6 1/2 | 15/7/2003 | 104.188 | 5.9121% | 6.9477 | 1.035197 |
| DBR | 6 | 15/9/2003 | 101.152 | 5.8390% | 7.2093 | 0.999674 |

The bonds above are also listed in order of relative price, with the cheapest to deliver at the top of the list. Again, the relatively higher coupon and relatively shorter duration are the predominant factors.

Determining which bond is the cheapest to deliver requires an understanding of where bond futures prices come from. To understand bond futures prices requires an understanding of cost of carry, repo rates and basis. Calculating the cheapest to deliver is based on these factors.

### Cost of Carry

The price of bond futures is tied to the price of bonds in the cash market. As cash prices rise, futures prices also rise.

In simple terms, the futures price is determined in the following way. By borrowing the money to buy a bond today in the cash market, a person can sell it forward in the futures market. For example he might buy the 6% *Bund* (the very last bond listed above under DBR, *Deutsche Bundesrepublik*, due 15 September 2003) at its market price of 101.152% of par.



He would be content to sell it forward as long as he can receive a price higher than the price he has just paid.

Since he has to borrow the money to buy it, he will have to pay interest on the loan. This interest is known as *cost of carry*, and it normally increases the price of anything in the future. In this simple example, the cost of the interest is added to the cash market price in calculating the break-even price at which the bond must sell in the futures market.

With commodities, such as wheat, corn, or oil, cost of carry includes other items, such as transportation costs (the literal meaning of cost of carry is the cost to transport a commodity from the fields to the buyer), insurance, storage, etc.

With interest rate futures contracts, the primary cost of carry is interest expense. Other costs are brokerage fees, margin costs, and other transaction costs. The most important of these is interest expense.

**Repo Rates**

*Repo rates* are the usual interest expense for financing the purchase of a bond. Meaning the rate on a *repurchase agreement*, the repo rate is the market rate at which one can borrow money in order to purchase a bond in the cash market.

Mechanically, using a repo (a repurchase agreement) is very simple. As long as he has a good name and a telephone, a party can call a bond dealer and ask to buy a bond. The dealer will ask how he wants to pay for it, and he can tell him he wishes to "*repo it*." This means he wishes to borrow the money from the dealer to buy the bond. He will leave the actual bond in the dealer's safekeeping, but he will be the owner, and he will owe the dealer interest on the loan at an agreed market rate. This rate is the repo rate. At the maturity of the loan, he can pay back the loan and the interest, and take possession of the bond. Or he can simply sell the bond back to the dealer at the current market price, and settle for the difference against the loan principal plus interest.

In this case, imagine he intends to take possession of the bond by repaying the loan and interest in full. He intends to do this because he is going to sell the bond through the futures market.

Repo markets do not exist for every kind of government bond. Although there is a very liquid repo market in the United States, the use of repos is only just beginning in Germany, for example. Where there is no liquid repo market, the repo rate is equal to the short-term interbank rate (Libor, Pibor, Fibor, Hibor, Sibor, etc.) for the period remaining through the futures market delivery date.

## Conversion Factors

When someone delivers a government bond to satisfy an existing short futures position, he sells it for the futures price agreed when the trade is done. There is only one futures price for each future delivery date, no matter which bond he intends to actually deliver.

Since the list of deliverable bonds contains a great diversity of coupon sizes and remaining maturities, the clearing house must use a conversion factor to make the prices of the different deliverable bonds more or less comparable.

The device used on most bond futures contracts, is to calculate a price at which the bond would yield the theoretical coupon rate of the futures contract, 8% for Treasury bond futures and 6% for *Bund* futures. This price is the bond's conversion factor. We will look at one example from each of the two contracts listed above.

## Calculating a Conversion Factor

| Issue: | Treasury | Treuhand |
|---|---|---|
| Maturity: | 15-Nov-2009 | 29-Jan-2003 |
| Coupon: | 11 3/4 | 7 1/8 |
| Market Price: | 156 19/32 | 107.363 |
| Conversion Factor: | 1.3298 | 1.076897 |
| Face Value: | $100,000 | DM250,000 |

### CBOT Treasury Bond Futures

The CBOT uses a convention for calculating the conversion factor which alters the actual maturity of the bond by forcing the maturity to an even number of calendar quarters. For example, the actual maturity of the 11 3/4% Treasury above is 15 November 2009 (this bond is callable at that date, and will otherwise mature in 2014). For purposes of calculating the conversion factor, the CBOT treats the first delivery day (the first business day of the delivery month) as the settlement date, 1 March 1994 for the March 1994 contract, and sets the bond's maturity at an even number of calendar quarters, as if it were 1 September 2009. Then a price is calculated which will yield the nominal coupon of 8%.

### Example

The conversion factor of 1.3298 for the Treasury bond is calculated as follows using both the HP12C and the HP19B.



| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 1) Clear financial registers and set | | [f][REG] | 0.0000 |
| accuracy to 4 digits | | [f][4] | |

| | | | |
|---|---|---|---|
| 2) Enter 8% YTM | 8 | [i] | 8.0000 |
| 3) Enter the coupon | 11.75 | [PMT] | 11.7500 |
| 4) Enter the first day of the delivery month for the March 1994 futures contract | 3.011994 | [ENTER] | 03.0120 |
| 5) Enter the adjusted maturity date and calculate the bond's price | 9.012009 | [f][PRICE] | 132.9784 |
| 6) Divide by 100 | 100 | [÷] | 1.3298 |

This is the conversion factor listed by the CBOT of 1.3298.

The same calculation can be performed using the HP19B:

| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 1) Set display to 4 digits | | [DISP] | TO SET #DECIMAL PLACES: PRESS {FIX} OR {ALL}. |
| | | [FIX] | TYPE #DIGITS (0-11); PRESS [INPUT] |
| | 4 | [INPUT] | |
| 2) Choose the financial menu | | [FIN] | SELECT A MENU |
| 3) Choose the bond menu | | [BOND] | A/A SEMIANNUAL |
| 4) Set the type of bond to actual/actual semi-annual | | [TYPE] | |
| | | [A/A] | |
| | | [SEMI] | A/A SEMIANNUAL |
| 5) Exit back to the bond menu | | [EXIT] | A/A SEMIANNUAL |
| 6) Enter the first day of the delivery month for the March 1994 futures contract | 3.011994 | [SETT] | SETT=03.01.1994 MON |
| 7) Enter the adjusted maturity date | 9.012009 | [MAT] | MAT=09.30.2009 TUE |

| | | | |
|---|---|---|---|
| 8) Enter the coupon | 11.75 | [CPN%] | CPN%=11.7500 |
| 9) Change menus | | [MORE] | |
| 10) Enter 8% YTM | 8 | [YLD%] | YLD%=8.0000 |
| 11) Solve for the price | | [PRICE] | PRICE=132.9784 |
| 12) Divide by 100 | 100 | [÷] | 1.3298 |

The CBOT actually uses a formula which duplicates the bond price calculation of the HP19B. The formula follows:

$$CF = \frac{\left[\dfrac{C}{2} + \dfrac{C}{Y} \times \left[1 - \dfrac{1}{\left(1+\dfrac{Y}{2}\right)^{2\times N}}\right] + \dfrac{1}{\left(1+\dfrac{Y}{2}\right)^{2\times N}}\right]}{\left(1+\dfrac{Y}{2}\right)^{\frac{x}{6}}} - \left(\frac{C}{2}\right) \times \left(\frac{6-x}{6}\right)$$

CF = Conversion Factor
C = Bond coupon rate in decimal form
Y = Nominal futures coupon of 8% in decimal form
N = Complete years to call or maturity, such that 2×N = the number of complete semi-annual periods to maturity.
X = The number of months in excess of the whole N, rounded down to complete quarters. For example, if the maturity is 27 years, 8 months, then N = 27 and X = 6.

If X calculated in this way equals 9, then 2×N should be set to 2×N+1 and X should be set to 3. For example if the maturity is 27 years, 11 months, then 2×N = 2×N+1 = 55 and X = 3.

**LIFFE *Bund* Futures**
LIFFE (and DTB) also uses a convention for calculating the conversion factor which alters the actual maturity of the bond by bringing it forward to the previous even 10th of the month. For example, the actual maturity of the 7 1/8% *Treuhand* above is 29 January 2003. For purposes of calculating the conversion factor, LIFFE sets the bond's maturity at 10 January 2003. Then a price is calculated which will yield the nominal coupon of 6%.

Unfortunately the HP12C cannot be used to calculate the price of a bond with annual coupons (without doing some programming), but the HP19B handles this very easily. First set the calculator to show 6 decimals, as that is the accuracy of LIFFE's conversion factors.

| | *Value* | *Key* | *Display* |
|---|---|---|---|
| Choose display | | [DISP] | TO SET #DECIMAL PLACES: PRESS {FIX} OR {ALL}. |
| Choose fix | | [FIX] | TYPE #DIGITS (0-11); PRESS [INPUT] |
| Set accuracy to 6 digits | 6 | [INPUT] | |
| 1) Choose the financial menu | | [FIN] | SELECT A MENU |
| 2) Choose the bond menu | | [BOND] | A/A SEMIANNUAL |
| 3) Set the type of bond to 30/360 annual | | [TYPE] [360] [ANN] | 30/360 ANNUAL |
| 4) Exit back to the bond menu | | [EXIT] | 30/360 ANNUAL |
| 5) Enter the delivery date for the December 1993 futures contract | 12.101993 | [SETT] | SETT=12.10.1993 FRI |
| 6) Enter the adjusted maturity date | 1.102003 | [MAT] | MAT=01.10.2003 FRI |
| 7) Enter the coupon | 7.125 | [CPN%] | CPN%=7.125000 |
| 8) Change menus | | [MORE] | |
| 9) Enter 6% YTM | 6 | [YLD%] | YLD%=6.000000 |
| 10) Solve for the price | | [PRICE] | PRICE=107.689678 |
| 11) Divide by 100 | 100 | [÷] | 1.076897 |

This is the conversion factor listed for this bond.

**Using a Conversion Factor**

When a bond is tendered to the exchange, the *invoice price* is calculated as the futures settlement price multiplied by the conversion factor, plus accrued interest. If the above *Treuhand* bond with a face value of DM250,000 were delivered to the exchange on 10 December 1993 at a futures price of 99.86, for example, the delivery proceeds to the seller would be DM268,847, before considering the accrued interest:

*Example*

$$\text{Futures Price} \times \text{Conversion Factor} = \text{Invoice Price}$$

$$99.86 \times 1.076897 = 107.538934$$

$$107.538934\% \times DM250,000 = DM268,847$$

Adding the accrued interest from the previous coupon date (actually the issue date) of 29 January 1993 through the futures delivery date of 10 December 1993, to this amount gives us the total amount of the delivery proceeds:

Using the HP19B we can calculate the accrued interest owing the seller:

| | ***Value*** | ***Key*** | ***Display*** |
|---|---|---|---|
| 1) Choose the financial menu | | [FIN] | SELECT A MENU |
| 2) Choose the bond menu | | [BOND] | A/A SEMIANNUAL |
| 3) Set the type of bond to | | [TYPE] | |
| 30/360 annual | | [360] | |
| | | [ANN] | 30/360 ANNUAL |
| 4) Exit back to the bond menu | | [EXIT] | 30/360 ANNUAL |
| 5) Enter the delivery date for the December 1993 futures contract | 12.101993 | [SETT] | SETT=12.10.1993 FRI |
| 6) Enter the actual maturity date | 1.292003 | [MAT] | MAT=01.29.2003 THU |
| 7) Enter the coupon | 7.125 | [CPN%] | CPN%=7.125000 |
| 8) Change menus | | [MORE] | |

| | | | |
|---|---|---|---|
| 9) Solve for the accrued interest | | [ACCRU] | ACCRU=6.155208 |
| 10) Divide by 100 | 100 | [÷] | 0.061552 |
| 11) Multiply by the face value | 250000 | [×] | 15,388.020833 |

With accrued interest of DM15,388 the full delivery proceeds come to:

| | ***Value*** | ***Key*** | ***Display*** |
|---|---|---|---|
| 1) Multiply the current futures price times | 99.86 | [×] | 99.860000× |
| :2) The conversion factor divided by | 1.076897 | ÷ | 107.538934÷ |
| 3) 100 times | 100 | × | 1.075389× |
| 4) The face value plus | 250000 | + | 268,847.336050+ |
| 5) Accrued interest | 15388.02 | = | 284,235.356050 |

The total delivery proceeds to the seller are DM284,235.36.

**Basis**

*Basis* is the difference between the price of a bond in the cash market and the price of the same bond in the futures market.

Basis comes in two forms: *net basis* and *gross basis*. Net basis refers to the difference in the price of the same bond in both markets, taking into consideration accrued interest, financing charges (repo rates), and any coupons received. Gross basis ignores all these financing costs and simply compares the two prices.

**Cash and Carry Arbitrage**

In order to understand basis, we will analyze a transaction wherein we buy a bond in the cash market and sell it in the futures market. Buying a bond in the cash market and selling it in the futures market, hoping to lock in a risk-free gain, is known as *cash and carry arbitrage*.

*Example*

The December 1993 *Bund* futures contract on LIFFE has a price of 99.86. Analysis of the cash and carry arbitrage using the bond at the top of the list of deliverable *Bunds* follows:

## December 1993 *Bund* Futures

| | |
|---|---|
| Settlement Date: | 22-Nov-93 |
| Futures Price: | 99.86 |
| Delivery Date: | 10-Dec-93 |
| Days: | 18 |
| **Cash Market:** | |
| Issue: | Treuhand |
| Maturity: | 29-Jan-2003 |
| Coupon: | 7 1/8 |
| Market Price: | 107.363 |
| Conversion Factor: | 1.076897 |
| Face Value: | DM250,000 |

The cash market price of the bond is calculated as follows. This calculation cannot be performed on the HP12C calculator, as the bond pays an annual coupon and the HP12C bond program only works for semi-annual U.S Treasury bonds. It is performed on the HP19B in the same manner as shown above in calculating the accrued interest and total price the futures exchange will pay:

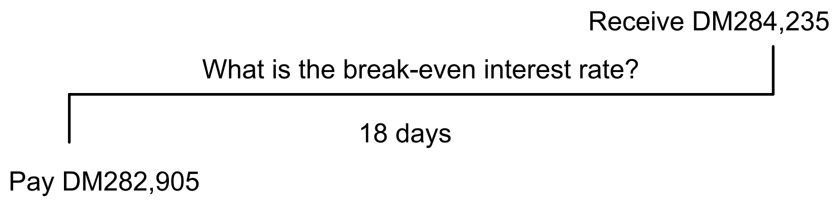
| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 1) Choose the financial menu | | [FIN] | SELECT A MENU |
| 2) Choose the bond menu | | [BOND] | A/A SEMIANNUAL |
| 3) Set the type of bond to | | [TYPE] | |
| 30/360 annual | | [360] | |
| | | [ANN] | 30/360 ANNUAL |
| 4) Exit back to the bond menu | | [EXIT] | 30/360 ANNUAL |
| 5) Enter the settlement date | 11.221993 | [SETT] | SETT=11.22.1993 MON |
| 6) Enter the maturity date | 1.292003 | [MAT] | MAT=01.29.2003 THU |
| 7) Enter the coupon | 7.125 | [CPN%] | CPN%=7.125000 |
| 8) Change menus | | [MORE] | |
| 9) Enter the price | 107.363 | [PRICE] | PRICE=107.363000 |

| | | | |
|---|---|---|---|
| 10) Add | | [+] | |
| 11) Accrued interest | | [ACCRU] | 107.363000+5.798958 |
| 12) Add the price and accrued interest together | | [=] | 113.161958 |
| 13) Divide by | | [÷] | 113.161958÷ |
| 14) 100 times | 100 | [×] | 1.131620× |
| 15) Face value | 250000 | [=] | 282,904.895832 |

The delivery proceeds are what the bond is worth on the delivery date through taking a short position in the futures market. This is exactly the same calculation already performed above on page 59, to calculate the total proceeds available on delivery of this bond. The delivery proceeds calculated above came to DM284,235.36.

Is there profit from this arbitrage? It depends on the financing cost. The break-even financing cost is the interest rate which makes the money to be repaid on the delivery date equal to the proceeds from the clearing house upon tendering the bonds.

This can be seen on a time line as follows:




The break-even interest rate is calculated as follows:

| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 1) Choose the financial menu | | [FIN] | SELECT A MENU |
| 2) Choose the time value of money menu | | [TVM] | 1 PMTS/YR: END MODE |
| 3) Enter the number of periods | 1 | [N] | N=1.0000 |
| 4) Enter the present value as a negative number | 282905 | [+/–] [PV] | PV=–282,905.0000 |
| 5) Enter the payments | 0 | [PMT] | PMT=0.0000 |

| | | | |
|---|---|---|---|
| 6) Enter the future value | 284235 | [FV] | FV=284,235.0000 |
| 7) Calculate the interest rate | | [I%YR] | I%YR=0.4701 |
| 8) Times | | [×] | 0.4701× |
| 9) 360 divided by | 360 | [÷] | 169.2441÷ |
| 10) The actual days | 18 | [=] | 9.4024 |

This is the break-even financing rate: 9.40%.

If the financing rate is lower than the break-even rate, buying the bond in the cash market and selling it in the futures market is a profitable arbitrage. With the repo rate at a level of 6.50%, this arbitrage would actually be profitable. Normally this is not the case.

The impled repo rate is usually less than the cash repo rate. Why? Because the seller of the future has certain options when delivering the bond. The seller can deliver the bond anytime during the month. Also the seller can deliver after the market closes. He can take advantage of economic news after the futures markets have closed, although this is less of a benefit to the seller now that the CBOT has night sessions. Also the seller has some timing options between actually notifying the exchange and delivering the bonds. These options are normally reflected in a lower futures price for the seller and hence a lower repo rate.

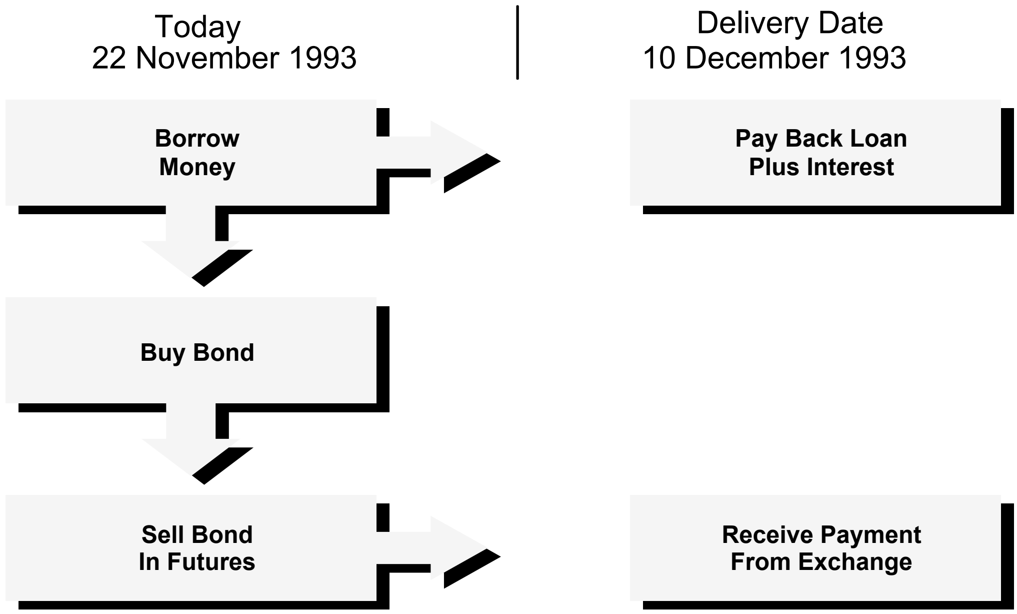
**Net Basis**

*Net basis*, also called *value basis*, is the difference in the two prices calculated above, expressed in the form of price points.

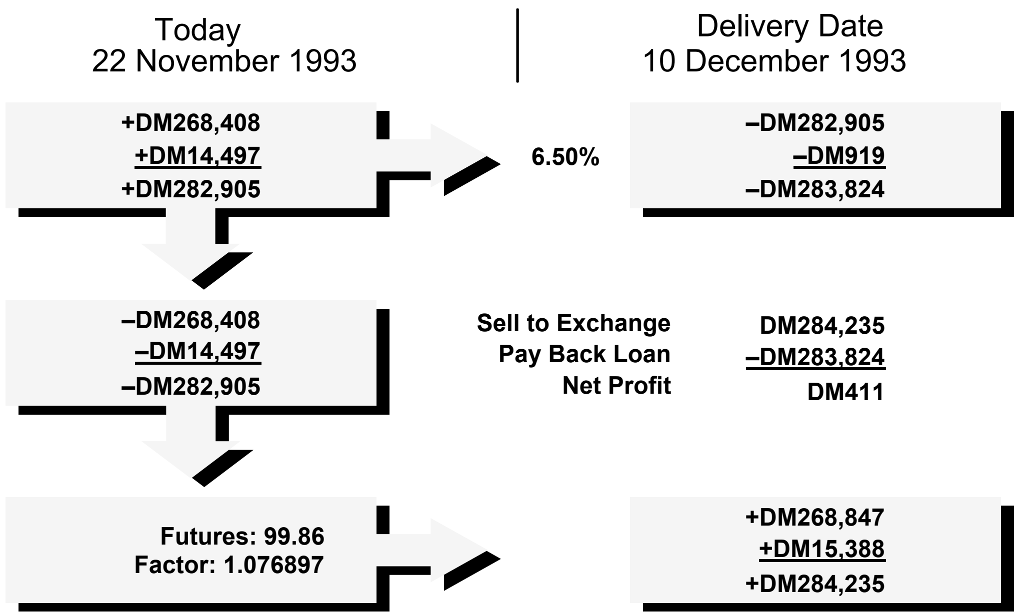
Net basis shows the profit or loss which would result from the cash and carry arbitrage financed at the market repo rate.

Comparing the financed price of buying the bond today and paying interest on the loan at the repo rate, to the total proceeds on delivery against a short position in the futures, calculates the net basis.

Here is a picture of the cash and carry arbitrage:




Putting amounts to the steps above shows the possible arbitrage:






In this case, profit amounts to DM411, which at DM25 per basis point of price (the tick value) represents net basis of 16.44 *basis points*. These basis points are expressed in terms of *price*, and should not be confused with basis points of interest rates.

$$\begin{array}{rr}
+\text{Futures Price} & +\text{DM}268{,}847 \\
+\text{Accrued Interest} & +\text{DM}15{,}388 \\
-\text{Cash Price} & -\text{DM}268{,}408 \\
-\text{Accrued Interest} & -\text{DM}14{,}497 \\
-\text{Repo Interest} & -\text{DM}919 \\
\hline
\text{Net Basis} & = \quad \text{DM}411
\end{array}$$

$$\text{Net Basis} = \frac{411}{25} = 16.44 \text{ basis points of price}$$

Comparing the gross price in the cash market to the gross price in the futures market gives us the *gross basis*:

$$\begin{array}{rr}
+\text{Futures Price} & +\text{DM}268{,}847 \\
-\text{Cash Price} & -\text{DM}268{,}408 \\
\hline
\text{Gross Basis} & = \quad \text{DM}439
\end{array}$$

$$\text{Gross Basis} = \frac{439}{25} = 17.56 \text{ basis points of price}$$

**Cheapest to Deliver**

We can now define the cheapest to deliver bond directly: it is that bond with the highest break-even financing cost, or implied repo. This is often also the bond with the highest net basis, meaning the bond with the biggest positive difference between the futures price and the cash market price. We are using positive basis to mean that the price in the futures market is **higher** than the price in the cash market.

Using this logic, we can now compare the deliverable bonds and see the implied repo rate as the deciding factor for the cheapest to deliver:

| Bond | Coupon | Maturity | Price | Conversion Factor | Net Basis | Implied Repo | Gross Basis |
|---|---|---|---|---|---|---|---|
| THA | 7 1/8 | 29/1/2003 | 107.363 | 1.076897 | 16.4 | 9.41% | 17.6 |
| THA | 6 7/8 | 11/6/2003 | 105.894 | 1.061493 | 9.6 | 8.27% | 10.7 |
| THA | 7 3/8 | 2/12/2002 | 109.050 | 1.092700 | 6.9 | 7.68% | 6.7 |
| THA | 7 3/4 | 1/10/2002 | 111.613 | 1.116075 | -14.1 | 4.01% | -16.2 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| THA | 6 1/2 | 23/4/2003 | 103.425 | 1.034535 | -14.0 | 3.89% | -11.6 |
| THA | 6 5/8 | 9/7/2003 | 104.363 | 1.043798 | -14.5 | 3.78% | -12.9 |
| DBR | 7 1/8 | 20/12/2002 | 107.956 | 1.076519 | -47.1 | -1.72% | -45.5 |
| THA | 6 | 12/11/2003 | 100.238 | 0.999868 | -41.7 | -1.82% | -39.1 |
| DBR | 7 1/4 | 21/10/2002 | 108.800 | 1.083526 | -59.2 | -4.32% | -59.9 |
| DBR | 6 3/4 | 22/4/2003 | 105.750 | 1.051997 | -71.7 | -6.57% | -69.8 |
| DBR | 8 | 22/7/2002 | 113.706 | 1.130619 | -78.1 | -6.92% | -80.2 |
| DBR | 6 1/2 | 15/7/2003 | 104.188 | 1.035197 | -83.4 | -9.17% | -81.3 |
| DBR | 6 | 15/9/2003 | 101.152 | 0.999674 | -135.7 | -20.04% | -132.5 |

Summarizing, we can observe that the 7 1/8% *Treuhand* due 29 January 2003 is the cheapest to deliver into the December *Bund* futures on LIFFE and DTB because it has the highest implied repo rate of 9.41%, and because it has the most favorable net basis of 16.4 basis points.

- The break-even financing rate for the cash and carry arbitrage is 9.41%. Since the actual market repo rate is 6.50%, the cash and carry arbitrage would be profitable.
- Buying it in the cash market and selling it in the futures market at 99.86 would yield a profit equal to 0.164% of the face value.

## Treasury Bond Futures Cheapest to Deliver

The same logic is true with Treasury bond futures. The list below shows some of the deliverable bonds into the March 1994 Treasury bond futures contract on the CBOT, ranked in order of the implied repo rate:

| Bond | Coupon | Maturity | Price | C. Factor | Net Basis | Implied Repo | Gross Basis |
|---|---|---|---|---|---|---|---|
| Treasury | 11 3/4 | 15/11/2009 | 156 19/32 | 1.3298 | -15.8 | 2.56% | -92.1 |
| Treasury | 13 1/4 | 15/5/2009 | 171 13/32 | 1.4539 | -18.9 | 2.48% | -107.0 |
| Treasury | 12 1/2 | 15/8/2009 | 164 3/32 | 1.3921 | -21.5 | 2.33% | -101.6 |
| Treasury | 11 1/4 | 15/2/2015 | 156 13/32 | 1.3262 | -30.8 | 1.79% | -99.4 |
| Treasury | 10 5/8 | 15/8/2015 | 149 16/32 | 1.2659 | -37.4 | 1.35% | -101.4 |
| Treasury | 9 7/8 | 15/11/2015 | 140 25/32 | 1.1910 | -39.1 | 1.09% | -99.5 |
| Treasury | 9 1/4 | 15/2/2016 | 133 13/32 | 1.1277 | -43.2 | 0.75% | -97.6 |
| Treasury | 8 3/4 | 15/5/2017 | 127 28/32 | 1.0783 | -51.1 | 0.07% | -103.4 |
| Treasury | 8 7/8 | 15/8/2017 | 129 14/32 | 1.0915 | -52.9 | 0.05% | -104.5 |
| Treasury | 9 1/8 | 15/5/2018 | 132 29/32 | 1.1192 | -58.4 | -0.26% | -113.1 |

The 11 3/4% coupon due 15 November 2009 is the cheapest to deliver (CTD) because it has the highest break-even financing cost, or implied repo rate, of 2.56%. If the maximum financing cost of a cash purchase is at a rate of 2.56%, a cash and carry arbitrage exactly breaks even.

With the market repo rate at 3.40% through the futures delivery date of 31 March 1994, the cash and carry arbitrage would lose money. How much? 15.8/32 (i.e. 0.49375) of the face value of the trade. The net basis is how much gain or lose exists through a cash and carry arbitrage, expressed in 32nds of a percent of face value (like the price quote on the Treasury bond and the futures contract itself).

To calculate the implied repo and net basis, keep in mind that the prices are all quoted in 32nds.

## Implied Repo

| | |
|---|---|
| **Relevant Rates and Futures Price on:** | **16-Nov-93** |
| June Bund Futures Contract: | |
| Price: | 115 19/32 |
| Delivery Date: | 31-Mar-94 |
| Days: | 135 |

| **Cash Market** | |
|---|---|
| Issue: | Treasury |
| Maturity: | 15-Nov-09 |
| Coupon: | 11 3/4 |
| Market Price: | 156 19/32 |
| Conversion Factor: | 1.3298 |
| Face Value: | $100,000.00 |

First we must buy the bond in the cash market:

| | |
|---|---|
| Cash Market Price: | 156 19/32 |
| Times Face Value: | $100,000.00 |
| Buy the Bund today and pay: | ($156,593.75) |
| Plus Accrued Interest: | ($32.46) |
| TOTAL Price | ($156,626.21) |

To calculate the accrued interest using the HP12C, you can do the following, taking into consideration that the last coupon was paid yesterday:

| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 1) Clear financial registers and set accuracy to 4 digits | | [f][REG] [f][4] | 0.0000 |
| 2) Enter the first day of the current coupon period | 11.151993 | [ENTER] | 11.1520 |
| 3) Enter the settlement date and calculate the number of days | 11.161993 | [g][ΔDYS] | 1.0000 |
| 4) Store for future use | | [STORE][1] | 1.0000 |
| 5) Enter the first day of the current coupon period | 11.151993 | [ENTER] | 11.1520 |
| 6) Enter the last day of the current coupon period and calculate the number of days | 5.151994 | [g][ΔDYS] | 181.000 |
| 7) Divide into 1 | | [1/*x*] | 0.0055 |

| | | | |
|---|---|---|---|
| 8) Multiply by the number of days stored above | | [RCL][1][×] | 0.0055 |
| 9) Multiply by the coupon | .1175 | [×] | 0.0006 |
| 10) Divide by 2 for the semi-annual coupon | 2 | [÷] | 0.0003 |
| 11) Multiply by the bond's face value | 100000 | [×] | 32.4586 |

The accrued interest amount is $32.46, as stated above.

Next sell the bond to the exchange and calculate the proceeds on the delivery date:

| | |
|---|---|
| Current Futures Price: | 115 19/32 |
| Expressed as a decimal: | 115.59375 |
| Times Conversion Factor: | 153.7166 |
| Times Face Value: | $153,716.57 |
| Plus Accrued Interest: | $4,414.36 |
| TOTAL Price | $158,130.93 |

The accrued interest of $4,414.36 is calculated as above, using a settlement date of 31 March 1994 instead of 16 November 1993.

Now calculate the implied repo rate for the 135-day period from 16 November 1993 to 31 March 1994:

| | ***Value*** | ***Key*** | ***Display*** |
|---|---|---|---|
| 3) Enter the number of periods | 1 | [n] | 1.0000 |
| 4) Enter the present value as a negative number | 156626.21 | [CHS][PV] | –156,626.2100 |
| 5) Enter the payments | 0 | [PMT] | 0.0000 |
| 6) Enter the future value | 158130.93 | [FV] | 158,130.9300 |
| 7) Calculate the interest rate | | [i] | 0.9607 |
| 8) Multiply by 360 | 360 | [×] | 345.8548 |
| 9) Divide by the actual days | 135 | [÷] | 2.5619 |

The implied repo rate for the 11 3/4% Treasury due 15 November 2009 is 2.56%.

**Net Basis**

To calculate net basis compare the all-in price of the bond in the cash market, financed at the market repo rate of 3.40%, to the all-in price of the bond in the futures market.

The all-in price of the bond in the cash market: $156,626.21.

Calculate the FV of this price at a rate of 3.40% for 135 days, and then calculate the basis expressed in price points of 1/32% by dividing it by the tick value of $31.25:

| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 1) Enter the number of periods | 1 | [n] | 1.0000 |
| 2) Enter the present value as a negative number | 156626.21 | [CHS][PV] | –156,626.2100 |
| 3) Enter the payments | 0 | [PMT] | 0.0000 |
| 4) Input the interest rate | 3.4 | [ENTER] | 3.4000 |
| 5) Multiply by the actual days | 135 | [×] | 459.0000 |
| 6) Divide by 360 and enter | 360 | [÷][i] | 1.2750 |
| 7) Calculate the future value | | [FV] | 158,623.1942 |
| 8) Change the sign | | [CHS] | –158,623.1942 |
| 9) Subtract from the all-in futures proceeds to get the net basis in $ | 158,130.93 | [+] | –492.2642 |
| 10) Divide by the tick value of $31.25 to get the number of ticks | 31.25 | [÷] | –15.75 |

The net basis is –15.75 ticks, which means the price in the cash market financed at the repo rate is 16 basis points higher than the price in the futures market.

| | | |
|---|---|---|
| +Futures Price | | +$153,716.57 |
| +Accrued Interest | | +$4,414.36 |
| –Cash Price | | –$156,593.75 |
| –Accrued Interest | | –$32.46 |
| –Repo Interest | | –$1,996.98 |
| Net Basis | = | – $492.26 |

$$\text{Net Basis} = \frac{-492.26}{31.25} = -15.75 \text{ ticks}$$

**Gross Basis**

Comparing the gross price in the cash market to the gross price in the futures market gives us the *gross basis*:

$$\begin{array}{rr} +\text{Futures Price} & +\$153{,}716.57 \\ -\text{CashPrice} & -\$156{,}593.75 \\ \hline \text{Gross Basis} & = \ -\$2{,}877.18 \end{array}$$

$$\text{Gross Basis} = \frac{-2{,}877.18}{31.25} = -92.07 \text{ ticks}$$

## *Summary*

- The oldest and most liquid futures contract covering changes in the value of governments bonds is the U.S. Treasury Bond future.
- All bond future contracts have specific deliverable bonds and tick values equal to the smallest price change of the underlying bond.
- U.S. Treasury Bond Futures trade on the Chicago Board of Trade (CBOT) and the London International Financial Futures and Options Exchange (LIFFE).
- *Bund* futures trade on LIFFE and the Deutsche Termin Börse (DTB) exchanges.
- The cheapest to deliver bond (CTD) is the bond most likely to be delivered by the seller of a bond futures contract.
- The cheapest to deliver bond is the bond which the seller can purchase most cheaply in the cash market to deliver into the futures contract.
- The cheapest to deliver bond is, out of all the bonds which are currently deliverable into the futures contract, the one which has the greatest implied repo rate. This bond theoretically offers the investor the largest return if he were to buy it in the cash market today and sell it in the futures market in the future.
- In low interest rate environments, bonds with high coupons and short duration tend to be the cheapest to deliver bonds.
- In high interest rate environments, bonds with low coupons and long duration tend to be the cheapest to deliver bonds.
- Repo rates are the interest expense for financing a purchase of a bond.
- Cost of carry is the interest rate or repo rate which increases the price of the bond in the future.
- The conversion factor for a deliverable bond is the adjustment to the price with which the bond would yield the theoretical coupon for the futures contract (8% for Treasury futures and 6% for *Bund* futures).
- *Basis* or *Gross Basis* is the difference in price between a bond in the cash market and the price of the same bond in the futures market.
- *Net Basis* refers to the difference in price of the same bond in the cash market and futures market taking into consideration accrued interest, financing charges (repo rates) and coupons.

## D. *Hedging with Bond Futures*

### Hedge Ratio Calculations

In order to hedge bonds with bond futures, we have to know how many futures contracts to use to offset the risk in the underlying bonds. There are two basic approaches used by market practitioners, which are in fact related to each other. The first we will analyze compares the change in value of the underlying bond position for a 0.01% change in the fixed rate to a change in value in the futures contract for a 0.01% change in the rate. The ratio of change gives us a hedge ratio based on **the value of a basis point**.

The second approach is similar, but attempts to measure the relative change in the underlying position and the hedge by means of **modified duration**. Since modified duration is an index of price sensitivity to a change of rate, the results should be similar under the two methods.

We will use a simple portfolio of a single *Bund* to test the first method, and a small portfolio of Treasuries to test the second.

### Value of a Basis Point

The portfolio we will hedge consists of the following bond:

| Bond | Coupon | Maturity | Price | Yield | Face Value |
|---|---|---|---|---|---|
| *Bund* | 6 | 9/15/2003 | 101.152 | 5.8390% | DM250,000,000 |

Since we own this bond, we will have to *sell futures* to hedge it. We are using the futures market to sell the bond *synthetically*.

### Underlying Bond

To hedge this portfolio, we have to measure how much its value changes if the applicable yield moves by one basis point up or down. With a simple portfolio like this all we have to do is recalculate the price. In order to establish a good reference, let us load this bond into the calculator and calculate its yield. We also need to know the bond's actual PV, which is the sum of the market price and the accrued interest, the so-called *dirty price*. For hedging purposes, it is the dirty price which we are protecting.



| | ***Value*** | ***Key*** | ***Display*** |
|---|---|---|---|
| 1) Choose the financial menu | | [FIN] | SELECT A MENU |
| 2) Choose the bond menu | | [BOND] | A/A SEMIANNUAL |

| | | | |
|---|---|---|---|
| 3) Set the type of bond to 30/360 | | [TYPE] | |
| annual | | [360] | |
| | | [ANN] | 30/360 ANNUAL |
| 4) Exit back to the bond menu | | [EXIT] | 30/360 ANNUAL |
| 5) Enter the settlement date | 11.221993 | [SETT] | SETT=11.22.1993 MON |
| 6) Enter the maturity date | 9.152003 | [MAT] | MAT=09.15.2003 MON |
| 7) Enter the coupon | 6 | [CPN%] | CPN%=6.000000 |
| 8) Change menus | | [MORE] | |
| 9) Enter the price | 101.152 | [PRICE] | PRICE=101.152000 |
| 10) Solve for YTM | | [YLD%] | YLD%=5.838977 |
| 11) Solve for the accrued interest | | [ACCRU] | ACCRU=1.116667 |
| 12) Recall the market price | | [RCL] | PRICE=101.152000 |
| | | [PRICE] | |
| 13) Add it to the accrued interest | | [+] | 102.268667 |

This is the initial dirty price of the bond which we are going to protect.

Without clearing the calculator, we will calculate the new dirty price under both a 1 basis point rise and fall of the current market yield.

| | | | |
|---|---|---|---|
| 14) Recall the YTM | | [RCL] | |
| | | [YLD%] | YLD%=5.838977 |
| 15) Add 1 basis point to it | .01 | [+] | 5.848977 |
| 16) Enter as the new YTM | | [YLD%] | YLD%=5.848977 |
| 17) Solve for the new price | | [PRICE] | PRICE=101.078306 |
| 18) Solve for the accrued interest | | [ACCRU] | ACCRU=1.116667 |
| 19) Add to the market price | | [+] | 102.194972 |



This is the new PV under a 0.01% rise in the yield. Now we do it again, assuming the yield falls by 0.01%:

| | | | |
|---|---|---|---|
| 20) Recall the YTM | | [RCL] | |
| | | [YLD%] | YLD%=5.848977 |
| 21) Subtract 2 basis points from it | .02 | [–] | 5.828977 |
| 22) Enter as the new YTM | | [YLD%] | YLD%=5.828977 |
| 23) Solve for the new price | | [PRICE] | PRICE=101.225763 |
| 24) Solve for the accrued interest | | [ACCRU] | ACCRU=1.116667 |
| 25) Add to the market price | | [+] | 102.342430 |

We can now make a table of the results:

| Yield | Price | Change in Price | Average |
|---|---|---|---|
| 5.848977% | 102.194972 | – 0.073695 | |
| 5.838977% | 102.268667 | | 0.073729 |
| 5.828977% | 102.342430 | + 0.073763 | |

What the table tells us is that the price of the bond changes by 0.073729 each time the market yield changes by 0.01%. For our holdings of DM250,000,000, this means a change in value of:

$$\Delta PV = 0.073729\% \times DM250{,}000{,}000$$

$$\Delta PV = DM184{,}322.1525$$

The above equation is read, "*Delta* PV equals ..." and means "the change in the PV..."

This is the value of one basis point in the underlying bond position, DM184,322.15.

**Futures Contract**

The value of the futures contract will change based on two rate changes: the yield of the cheapest to deliver bond ("the CTD") and the repo rate. The repo rate is the primary source of basis in the CTD — assuming that the implied repo of the CTD stays relatively near the market repo, or that the net basis remains fairly small — and is notoriously difficult to hedge. Using bond futures it is not possible to hedge the basis — hence the origin of the term *basis risk* — but we can hedge changes in the yield of the CTD.



The CTD is itself another source of risk: can we be sure that the spread between the yield of the CTD and the yield of the bond(s) we wish to hedge will remain constant? If it does not, the changing spread introduces another source of risk into the hedge.

If we assume that the basis will remain constant and that changes in the market yield of the CTD will be mirrored basis point for basis point by changes in the yield of the bond(s) we wish to hedge, we can calculate a hedge ratio in the following manner.

First, we have to calculate how the changing yield of the CTD will affect its market value. This is essentially the same comparison we made above for the bond we own. Then we calculate a new futures price — assuming no change in the implied repo rate for the CTD. This gives us the value of a 0.01% change in the yield of the CTD on a single futures contract.

| Bond | Coupon | Maturity | Price | Yield | Modified Duration | Face Value |
|---|---|---|---|---|---|---|
| *Treuhand* | 7 1/8 | 1/29/2003 | 107.363 | 6.0521% | 6.3839 | DM250,000 |

| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 1) Choose the financial menu | | [FIN] | SELECT A MENU |
| 2) Choose the bond menu | | [BOND] | A/A SEMIANNUAL |
| 3) Set the type of bond to 30/360 | | [TYPE] | |
| annual | | [360] | |
| | | [ANN] | 30/360 ANNUAL |
| 4) Exit back to the bond menu | | [EXIT] | 30/360 ANNUAL |
| 5) Enter the settlement date | 11.221993 | [SETT] | SETT=11.22.1993 MON |
| 6) Enter the maturity date | 1.292003 | [MAT] | MAT=01.10.2003 WED |
| 7) Enter the coupon | 7.125 | [CPN%] | CPN%=7.125000 |
| 8) Change menus | | [MORE] | |
| 9) Enter the price | 107.363 | [PRICE] | PRICE=107.363000 |
| 10) Solve for YTM | | [YLD%] | YLD%=6.052064 |
| 11) Solve for the accrued interest | | [ACCRU] | ACCRU=5.798958 |
| 12) Recall the market price | | [RCL] | PRICE=107.363000 |
| | | [PRICE] | |
| 13) Add it to the accrued interest | | [+] | 113.161958 |

This is the initial dirty price of the cheapest to deliver bond.

Without clearing the calculator, we will calculate the new dirty price under both a 1 basis point rise and fall of the current market yield.

| | | | |
|---|---|---|---|
| 14) Recall the YTM | | [RCL] | |
| | | [YLD%] | YLD%=6.052064 |
| 15) Add 1 basis point to it | .01 | [+] | 6.062064 |
| 16) Enter as the new YTM | | [YLD%] | YLD%=6.062064 |
| 17) Solve for the new price | | [PRICE] | PRICE=107.290790 |
| 18) Solve for the accrued interest | | [ACCRU] | ACCRU=5.798958 |
| 19) Add to the market price | | [+] | 113.089748 |

This is the new PV under a 0.01% rise in the yield. Now we do it again, assuming the yield falls by 0.01%:

| | | | |
|---|---|---|---|
| 20) Recall the YTM | | [RCL] | |
| | | [YLD%] | YLD%=6.062064 |
| 21) Subtract 2 basis points from it | .02 | [–] | 6.042064 |
| 22) Enter as the new YTM | | [YLD%] | YLD%=6.042064 |
| 23) Solve for the new price | | [PRICE] | PRICE=107.435273 |
| 24) Solve for the accrued interest | | [ACCRU] | ACCRU=5.798958 |
| 25) Add to the market price | | [+] | 113.234232 |

We can now make a table of the results:

| Yield | Price | Change in Price | Average |
|---|---|---|---|
| 6.062064% | 113.089748 | – 0.072210 | |
| 6.052064% | 113.161958 | | 0.072242 |
| 6.042064% | 113.234232 | + 0.072274 | |

We will use the average price change of 0.072242 in the CTD for a 0.01% change in its yield. The future value of this price change at the implied repo rate of 9.41% (calculated earlier) for 18 days (the period through delivery), adjusted by the conversion factor, gives us the value of the 0.01% on the futures price. The effect of using the FV at the implied repo rate is very small (less than 1/3 of a basis point), so we will ignore it and make the equation simpler.

This relationship can be reduced to a fairly straightforward formula, if we assume that the face value of the CTD is equal to the face value of a single futures contract:

$$\frac{\Delta PV_F}{100} \times FV_F \times CF = \frac{\Delta PV_{CTD}}{100} \times FV_{CTD}$$

$$\Delta PV_F = \frac{\Delta PV_{CTD}}{CF}$$

$$\Delta PV_F = \frac{0.072242}{1.076897}$$

$$\Delta PV_F = 0.067083$$

**Where:**

$\Delta PV_F$ = Change in price of 1 futures contract
$FV_F$ = Face value of 1 futures contract
$CF$ = Conversion factor for the CTD
$\Delta PV_{CTD}$ = Change in price of the CTD
$FV_{CTD}$ = Face value of the CTD

The formula tells us that the price change we can expect in the futures contract for a 1 basis point change in the yield of the CTD is 0.067083, or 6.7083 ticks. Since each tick is worth DM25, and we have 6.7083 of them, the value of a 0.01% change in the yield of the CTD is DM167.71 per futures contract.

$$\Delta PV_F = 6.7083 \times DM25 = DM167.7075 \text{ per contract}$$

**Hedge Ratio Calculation**

Since we know the value of a 0.01% change in yields for both the underlying bond and for the futures contract, we can use the two values to calculate our hedge ratio:



$$\frac{DM184,322.15}{DM167.7075} = 1,099.07$$

This ratio tells us that we need to use 1,099 futures contracts to offset the change in PV on our portfolio from a 0.01% change in yields. This makes sense intuitively, as our cash position is roughly 1,000 times bigger than the face value of a single futures contract.

We need to use slightly more than 1,000 contracts because the PV of the CTD (and hence the futures) is less sensitive to changes in rates than is the PV of the newly issued (and longer duration) 6% *Bund*.

**Modified Duration-Based Hedge Ratio**

The above relationships can be stated a good deal more simply if we use modified duration to give us the value of the 0.01% change in yield. Since that is what modified duration is, i.e. an index of price sensitivity to changes in yields, this is a very reasonable approach. It is also easier!

We can summarize the above calculations into a single, fairly straightforward hedge ratio. The derivation of this ratio is beyond the scope of this self-instructional guide, but we can use it without having to know how to derive it. The logic is very similar to that followed above, as we are using the relative price sensitivity of the CTD (as the source of changes in the futures price) and the bonds we own to determine how many futures contracts to sell.

The basic relationship is as follows:

$$PV_U \times MD_U \times \Delta i_U = -\#C \times PV_{CTD} \times MD_{CTD} \times \frac{FV_F}{CF} \times \Delta i_{CTD}$$

**Where:**

$PV_U$ = Market value of the underlying bond or portfolio of bonds
$MD_U$ = Modified duration of the underlying bond or portfolio of bonds
$\Delta i_U$ = Change in yield of the underlying bond or portfolio of bonds
$\#C$ = Number of futures contracts
$PV_{CTD}$ = Market value of the CTD, expressed as a decimal
$MD_{CTD}$ = Modified duration of the CTD
$FV_F$ = Face value of the futures contract
$CF$ = Conversion factor for the CTD
$\Delta i_{CTD}$ = Change in yield of the CTD



The above formula can be read, “The market value of the underlying times the modified duration of the underlying times the expected change in yield of the underlying is to be offset by a change in the futures position equal to the number of futures contracts times the market value of the cheapest to deliver times the modified duration of the CTD times

the face value of one futures contract divided by the conversion factor of the CTD times the expected change in yield of the CTD."

If we wish to know how many futures contracts to buy or sell, we must solve the above equation for #C:

$$\#C = -\frac{PV_U}{PV_{CTD}} \times \frac{MD_U}{MD_{CTD}} \times \frac{CF}{FV_F} \times \frac{\Delta i_U}{\Delta i_{CTD}}$$

Note that we are hedging the market value of the underlying position, i.e. its dirty price times the face value of our holdings. We will also assume that the yield spread between the CTD *Treuhand* and the *Bund* we own remains constant, so that the ratio of changes in the interest rates is 1 to 1. We might make any assumption we wish about this, actually, but the level of the spread is hard to predict.

We can use the above equation to solve for the number of contracts we need to sell in order to hedge our position in the new 6% *Bund* by plugging in the numbers:

$$\#C = -\frac{250{,}000{,}000 \times 102.268667\%}{113.161958\%} \times \frac{7.2093}{6.3839} \times \frac{1.076897}{250{,}000} \times \frac{0.01\%}{0.01\%}$$

$$\#C = -1{,}099.06$$

Again, we need to sell 1,099 December *Bund* futures contracts to protect our holdings of DM250,000,000 of the 6% *Bund* due 15 September 2003 from rising rates.

This is the same number of contracts we calculated above using the value of a basis point method. It should be, as modified duration is an index of price change given a change in rates.

We might also summarize the assumptions we have made, or the risks we are taking in hedging this position:

1. We assume no change in the yield spread between the CTD 7 1/8% *Treuhand* and the 6% *Bund* we own, currently 0.2131%.

2. We assume no change in the net basis or repo rate for the CTD, currently 0.164 and 9.41% respectively.

3. We also assume that the CTD bond does not change.

If all these assumptions are good, our hedge will work pretty well.

**Adjusting the Duration of the Underlying Position**

In the example above, we used the futures market to offset all of the duration of the underlying position.

In a similar fashion, we can use the futures to adjust the duration of our portfolio to a level we wish to target. To do so, we need only consider the *weighted average duration* of the portfolio we wish to achieve.

An important note is that futures cannot be used to extend the duration of a portfolio out past the duration of the CTD. By selling futures against a portfolio of bonds we own, however, we can use them to decrease the duration of the portfolio.

However much of our existing position we offset using futures, we reduce the duration to 0 for that amount of the position.

Above, for example, we brought the duration of our net position, owning the 6% *Bund* and selling 1,099 December 1993 *Bund* futures contracts, to 0.

We can express this relationship in a formula as follows:

$$\text{Hedge\%} \times MD_{Hedge} + (1 - \text{Hedge\%}) \times MD_{U} = MD_{Target}$$

Whatever percent of the underlying position we hedge, we reduce its duration, the $MD_{Hedge}$ above, to 0. If we select a target duration, we can solve for the Hedge% we need to achieve it. This is a function of the duration of the underlying position. We can therefore simplify the above equation as follows:

$$\text{Hedge\%} = \frac{MD_{U} - MD_{Target}}{MD_{U}}$$

This equation shows us that if we wish to achieve a target duration of 0, we have to hedge 100% of our underlying position:

$$\text{Hedge\%} = \frac{7.2093 - 0}{7.2093} = 100\%$$

If we wished to decrease the duration of the underlying position to 3, for example, we would have to hedge 58.39% of the position:

$$\text{Hedge\%} = \frac{7.2093 - 3}{7.2093} = 58.3871\%$$

We know from above that to hedge 100% of our underlying position, we would have to sell 1,099 contracts. Therefore to hedge 58.39% of our position, we have to sell 641 contracts:

$$58.3871\% \times 1{,}099 = 641.67$$

It should be noted that we are not actually changing the duration of the portfolio in terms of the tenor of the rates to which we have exposure. Rather, what we are doing is making smaller the impact of a change in the same rate on our portfolio. It is best to consider this decrease in duration not in terms of time, or years, but rather in terms of sensitivity. If we begin with an underlying position that is 100% sensitive to a change in the market yields affecting it, we can reduce the index of sensitivity to 0% by fully offsetting the underlying position through the sale of futures contracts.

In the example above, we have reduced the sensitivity of our position to changing rates from 100% to 41.61% (100% – 58.39%) by reducing the duration to 3.

*Summary*

- A hedge ratio determines how many futures contracts to use to offset the risk of an underlying bond.
- The change in the value of a bond for a 1 basis point change in rates is known as the value of a basis point.
- Either modified duration analysis or value of a basis point analysis can be used to determine the hedge ratio. The results should be similar.
- The dirty price of a bond is the market value plus any accrued interest. Hedging should control risk for the dirty price of a bond.
- The value of a futures price will change based on two rates - the yield of the cheapest to deliver bond and the repo rate.
- Changes due to the repo rate are difficult to hedge. However the changes in the yield of the cheapest to deliver bond can be hedged.
- If the spread between the yield on the cheapest to deliver bond and the bond being hedged does not remain constant, this will add another source of risk to the hedge.
- If the cheapest to deliver bond changes the hedge will need to be adjusted.
- Futures can be used to decrease the duration of a portfolio but futures cannot be used to increase the duration of a portfolio out past the duration of the cheapest to deliver.

## *Exercises*

1. Please calculate the break-even financing cost for buying the 6% *Bund* due 15 September 2003 and selling it into the December 1993 *Bund* futures contract.

2. Please calculate the implied repo rate, net basis and gross basis for the same bond.

3. Please calculate how many March 1994 Treasury bond futures contracts you need to sell to hedge all price change in the following portfolio of Treasury bonds:

| Bond | Coupon | Maturity | Price | Yield | Modified Duration | Face Value | Market Value |
|---|---|---|---|---|---|---|---|
| Treasury | 6 1/4 | 15-Aug-2023 | 101 5/32 | 6.1638% | 13.3184 | $75,000,000 | $77,051,800.27 |
| Treasury | 8 1/2 | 15-Feb-2020 | 125 21/32 | 6.4569% | 11.7578 | $55,000,000 | $70,292,391.30 |
| Treasury | 12 1/2 | 15-Aug-2009 | 164 3/32 | 6.1050% | 8.5638 | $25,000,000 | $41,813,179.35 |
| Portfolio | | | | | 11.6875 | $155,000,000 | $189,157,370.92 |
| | | | | | | | |
| CTD | 11 3/4 | 15-Nov-2009 | 156 19/32 | 6.1436% | 8.8916 | | |

4. How many contracts must you sell if you wish to decrease the duration of the portfolio owned to 5 years?

5. Please calculate the conversion factor for the following bond deliverable into the March 1994 Treasury bond futures contract:

| **Cash Market** | |
|---:|---:|
| Issue: | Treasury |
| Maturity: | 15-May-2018 |
| Settlement: | 16-Nov-1993 |
| Coupon: | 9 1/8 |
| Market Price: | 132 29/32 |
| Face Value: | $100,000.00 |

| **Futures Market** | |
|---:|---:|
| March 1994 Treasury Bond Futures Contract Price: | 115 19/32 |
| Settlement | 16-Nov-1993 |
| Delivery Date: | 31-Mar-1994 |
| Days: | 135 |

## APPENDIX I. EXCHANGE DIRECTORY

This exhibit contains a summary of the leading interest rate futures exchanges. It includes information on the interest rate futures contracts traded as of December 1993.

## AUSTRALIA

**SYDNEY FUTURES EXCHANGE — SFE**
*(Sydney, Australia)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| 90-day Bank Bills | A$500,000 | 0.01% ≈ A$12.00 (variable) | Mar Jun Sep Dec |
| 3-year T-bonds | A$100,000 | 0.01% = A$25 | Mar Jun Sep Dec |
| 10-year T-bonds | A$100,000 | 0.005% = A$25.00 | Mar Jun Sep Dec |

## BELGIUM

**Belgian Futures and Options Exchange — BELFOX**
*(Brussels, Belgium)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| Belgian Gov. Bonds | BEF2,500,000 | 0.01 pt. = BEF250 | Mar Jun Sep Dec |
| BIBOR 3-Month | BEF25,000,000 | 0.01% = BEF2,500 | Mar Jun Sep Dec |

## BRAZIL

**BOLSA BRASILEIRA DE FUTUROS**
*(Rio de Janeiro, Brazil)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| BTN (T-bill) | BTN5,000 | Cr$0.01 | All Months |

**BOLSA DE MERCADORIAS & FUTUROS — BM&F**
*(São Paulo, Brazil)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| 1-Day Interbank Deposits | Cr$100,000,000 | 1 pt. = Cr$1,000 | All Months |
| 30-Day Interbank Deposits | Cr$100,000,000 | 1 pt. = Cr$1,000 | All Months |

## CANADA

**MONTREAL EXCHANGE — ME**
*(Montreal, Quebec, Canada)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| 1-Month Canadian BAs | C$3,000,000 | 0.01% = C$25 | First 6 months |
| 3-Month Canadian BAs | C$1,000,000 | 0.01% = C$25 | Mar Jun Sep Dec |
| 10-Year Gov. of Canada Bond | C$100,000 | 0.01 pt. = C$10 | Mar Jun Sep Dec |
| Gov. of Canada Bond | C$25,000 | 0.01 pt. = C$2.50 | Mar Jun Sep Dec: 3 months plus two |

## DENMARK

**COPENHAGEN STOCK EXCHANGE AND GUARANTEE FUND FOR DANISH OPTIO AND FUTURES — FUTOP**
*(Copenhagen, Denmark)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| Danish Gov. Bonds (series of contracts on specific issues) | DKr1,000,000 | 0.05 pt. = DKr500 | Mar Jun Sep Dec |
| Mortgage Credit Bonds 9% 2022 | DKr1,000,000 | 0.05 pt. = DKr500 | Mar Jun Sep Dec |

## FRANCE

**MARCHÉ À TERME INTERNATIONAL DE FRANCE — MATIF**
*(Paris, France)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| Long-Term Notional Bond | FFr500,000 | 0.02% = FFr100 | Mar Jun Sep Dec |
| 3-month PIBOR | FFr5,000,000 | 0.01% = FFr125 | Mar Jun Sep Dec |
| Long-Term Italian Bond | Lit100,000,000 | 1 pt. = Lit10,000 | Mar Jun Sep Dec |
| ECU Bond | ECU100,000 | 0.02 pt. = ECU20 | Mar Jun Sep Dec |

## GERMANY

**DEUTSCHE TERMINBÖRSE — DTB**
*(Frankfurt, Germany)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| Long-Term Gov Bond: *Bund* | DM250,000 | 0.01 = DM25.00 | Mar Jun Sep Dec |
| Medium-Term Gov Bond: *Bobl* | DM250,000 | 0.01 = DM25.00 | Mar Jun Sep Dec |
| 3-Month FIBOR (soon) | DM1,000,000 | 0.01% = DM25 | Mar Jun Sep Dec |
| Extra-Long-Term Gov Bond: Buxl (soon) | DM250,000 | 0.01 = DM25.00 | Mar Jun Sep Dec |

## HONG KONG

**HONG KONG FUTURES EXCHANGE LTD. — HKFE**
*(Hong Kong)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| 3-month HIBOR (inactive since 1991) | HK1,000,000 | 1 pt. = HK$25 | Mar Jun Sep Dec |

## IRELAND

**IRISH FUTURES & OPTIONS EXCHANGE — IFOX**
*(Dublin, Ireland)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| Long Gilt | IR£50,000 | IR£5 | Mar Jun Sep Dec |
| Short Gilt | IR£100,000 | IR£10 | Mar Jun Sep Dec |
| 3-month Interest Rate | IR£500,000 | IR£12.50 | Mar Jun Sep Dec |

## JAPAN

**TOKYO INTERNATIONAL FINANCIAL FUTURES EXCHANGE — TIFFE**
*(Tokyo, Japan)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| 3-month Euroyen | ¥1,000,000,000 | 0.01 pt. = ¥2,500 | Mar Jun Sep Dec |
| 3-month Eurodollar | $1,000,000 | 0.01 pt. = $25 | Mar Jun Sep Dec |
| 1-Year Euroyen | ¥100,000,000 | 0.01 pt. = ¥10,000 | Mar Jun Sep Dec |

**TOKYO STOCK EXCHANGE — TSE**
*(Tokyo, Japan)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| 10-yr. Japanese Gov. Bond | ¥100,000,000 | 0.01 pt. = ¥10,000 | Mar Jun Sep Dec |
| 20-yr. Japanese Gov. Bond | ¥100,000,000 | 0.01 pt. = ¥10,000 | Mar Jun Sep Dec |
| U.S. T-bond | $100,000 | 1/32 pt. = US$31.25 | Mar Jun Sep Dec |

## NETHERLANDS

**EUROPEAN OPTIONS EXCHANGE — EOE-Optiebeurs**
*(Amsterdam,, Netherlands)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| Dutch Gov. Bonds | DFl10,000 | DFl0.01 | Feb May Aug Nov |
| Notional Bond | DFl250,000 | DFl0.01 | Feb May Aug Nov |

**FINANCIELE TERMIJNMARKT AMSTERDAM N.V. — FTA**
*(Amsterdam, Netherlands)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| Guilder bonds | DFl250,000 | 0.01 pt. = DFl25 | Mar Jun Sep Dec |

## NEW ZEALAND

**NEW ZEALAND FUTURES & OPTIONS EXCHANGE — NZFOE**
*(Auckland, New Zealand)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| 90-day Bank Accepted Bills | NZ$500,000 | 0.01% ≈ NZ$12 (variable) | Mar Jun Sep Dec |
| 5-yr. Gov. Stock #2 | NZ$100,000 | 0.01 = NZ$10 | Mar Jun Sep Dec |
| 10-yr. Gov. Stock | NZ$100,000 | 0.01 = NZ$10 | Mar Jun Sep Dec |

## PHILIPPINES

**MANILLA INTERNATIONAL FUTURES EXCHANGE — MIFE**
*(Makati, The Philippines)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| Interest Rates | 10,000 pesos | 0.01% | Next 4 months |

## SINGAPORE

**SINGAPORE INTERNATIONAL MONETARY EXCHANGE — SIMEX**
*(Singapore)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| Eurodollar | $1,000,000 | 0.01 pt. = US$25 | Mar Jun Sep Dec |
| Euromark | DM1,000,000 | 0.01 pt. = DM25 | Mar Jun Sep Dec |
| Euroyen | ¥100,000,000 | 0.01 pt. = ¥2,500 | Mar Jun Sep Dec |

## SOUTH AFRICA

**SOUTH AFRICAN FUTURES EXCHANGE — SAFEX**
*(Johannesburg, South Africa)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| Short-Term Interest | 1,000,000 R | 0.01% = 25 R | Feb May Aug Nov |
| Long Bond | 100,000 R | 0.01 = 10 R | Feb May Aug Nov |

## SPAIN

**MERCADO DE FUTUROS FINANCIEROS S.A. — MEFF**
*(Barcelona, Spain)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| MIBOR | Ptas10,000,000 | 1 pt. = Ptas 250 | Mar Jun Sep Dec |
| 3-Year Notional Bond | Ptas 10,000,000 | 1 pt. = Ptas 250 | Mar Jun Sep Dec |
| 10-Year Notional Bonds | Ptas10,000,000 | 1 pt. = Ptas 1,000 | Mar Jun Sep Dec |

## SWEDEN

**STOCKHOLM OPTIONS MARKET — OM Stockholm**
*(Stockholm, Sweden)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| OMr5 Notional Bonds | SKr1,000,000 | 0.01 pt. | Mar Jun Sep Dec |
| OMR7 Notional Bonds | SKr1,000,000 | 0.01 pt. | Mar Jun Sep Dec |
| Notional T-bills | SKr1,000,000 | 0.01 pt. | Mar Jun Sep Dec |
| OMr10 Notional Bonds | SKr1,000,000 | 0.01 pt. | Mar Jun Sep Dec |
| MBB5 Mortgage Bonds | SKr1,000,000 | 0.01 pt. | Mar Jun Sep Dec |
| CT2 Mortgage Bonds | SKr1,000,000 | 0.01 pt. | Mar Jun Sep Dec |
| CT5 Mortgage Bonds | SKr1,000,000 | 0.01 pt. | Mar Jun Sep Dec |
| Stibor | SKr1,000,000 | 0.01 pt. | Mar Jun Sep Dec |
| OMSwap | SKr1,000,000 | 0.01 pt. | Mar Jun Sep Dec |
| SBAB 5 Mortgage | SKr1,000,000 | 0.01 pt. | Mar Jun Sep Dec |
| Interest Rate | SKr1,000,000 | 0.01 SEK | Mar Jun Sep Dec |
| OMr2 | SKr1,000,000 | 0.01 pt. | Mar Jun Sep Dec |

## SWITZERLAND

**SWISS OPTIONS AND FINANCIAL EXCHANGES — SOFFEX**
*(Dietikon, Switzerland)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| 3-Month Eurofranc | SFr1,000,000 | 0.01 pt. = SFr25 | Mar Jun Sep Dec |
| 5-year Swiss Franc Interest Rate | SFr100,000 | 0.01% = SFr10 | Next 4 months of Mar Jun Sep Dec |
| Swiss Gov. Bonds | SFr100,000 | 0.01% = SFr10 | Next 4 months of Mar Jun Sep Dec |

## UNITED KINGDOM

**LONDON INTERNATIONAL FINANCIAL FUTURES AND OPTIONS EXCHANGE — L**
*(London, England)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| 3-month Eurodollars | $1,000,000 | 0.01% = $25 | Mar Jun Sep Dec |
| 3-Month Sterling Interest Rate | £500,000 | 0.01% = £12.50 | Mar Jun Sep Dec |
| 3-month Euromark | DM1,000,000 | 0.01% = DM25 | Mar Jun Sep Dec |
| 3-month Euro Swiss | SFr1,000,000 | 0.01% = SFr25 | Mar Jun Sep Dec |
| 3-month Euro Lira Interest Rate | Lit1,000,000,000 | 0.01% = Lit25,000 | Mar Jun Sep Dec |
| 3-month ECU Interest Rate | ECU1,000,000 | 0.01% = ECU25 | Mar Jun Sep Dec |
| Long Gilt | £50,000 | £1/32 = £15.625 | Mar Jun Sep Dec |
| U.S. T-bonds | $100,000 | 1/32 pt. = US$31.25 | Mar Jun Sep Dec |

| | | | |
|---|---|---|---|
| Japanese Gov. Bond | ¥100,000,000 | 0.01 pt. = ¥10,000 | Mar Jun Sep Dec |
| German Gov. Bond: *Bund* | DM250,000 | 0.01 pt. = DM25 | Mar Jun Sep Dec |
| Medium-Term German Gov. Bonds: *Bobl* | DM250,000 | 0.01 pt. = DM25 | Mar Jun Sep Dec |
| Italian Gov. Bonds | Lit200,000,000 | 0.01 pt. = Lit20,000 | Mar Jun Sep Dec |

## UNITED STATES

**CHICAGO BOARD OF TRADE — CBOT**
*(Chicago, Illinois)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| U.S. Treasury Bonds | $100,000 | 1/32 pt. = $31.25 | Mar Jun Sep Dec |
| 10-yr. T-notes | $100,000 | 1/32 pt. = $31.25 | Mar Jun Sep Dec |
| 5-yr. T-notes | $100,000 | 1/64 pt. = $15.625 | Mar Jun Sep Dec |
| 2-yr. T-notes | $200,000 | 1/4 of 1/32 pt. = $15.625 | Mar Jun Sep Dec |
| 30-day Interest rate | $5,000,000 | 0.01% = $41.67 | All Months |
| Municipal Bond Index | $1,000 × Bond Buyer Index | 1/32 pt. = $31.25 | Mar Jun Sep Dec |
| **PROJECT A CONTRACTS** | | | |
| Zero Coupon Bond | $100,000 | 1/32 pt. = $31.25 | Mar Jun Sep Dec |
| Zero Coupon Note | $100,000 | 1/32 pt. = $31.25 | Mar Jun Sep Dec |

**CHICAGO MERCANTILE EXCHANGE (International Monetary Market Division) — I**
*(Chicago, Illinois)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| Treasury Bills | $1,000,000 | 1 pt. = $25 | Mar Jun Sep Dec |
| Eurodollar Time Deposit | $1,000,000 | 1 pt. = $25 | Mar Jun Sep Dec |
| 1-month LIBOR | $3,000,000 | 1 pt. = $25 | All Months |

**MIDAMERICA COMMODITY EXCHANGE — MidAm**
*(Chicago, Illinois)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| U.S. T-bonds | $50,000 | 1/32 pt. = $15.62 | Mar Jun Sep Dec |
| U.S. T-bills | $500,000 | 1 pt. = $12.50 | Mar Jun Sep Dec |
| U.S. T-notes | $50,000 | 1/32 pt. = $15.62 | Mar Jun Sep Dec |
| Eurodollar | $500,000 | 1 pt. = $12.50 | Mar Jun Sep Dec |

**FINANCIAL INSTRUMENT EXCHANGE — FINEX**
*(division of the New York Cotton Exchange — New York, New York)*

| Futures | Size | Tick Size | Months |
|---|---|---|---|
| 2-year T Auction Notes | $100 × basis points of yield | 0.005 pt. = $50 | All months |
| 5-year T Auction Notes | $100 × basis points of yield | 0.005 pt. = $50 | All months |

# Bond Futures

1. Please calculate the break-even financing cost for buying the 6% *Bund* due 15 September 2003 and selling it into the December 1993 *Bund* futures contract.

| | |
|---|---|
| **Relevant Rates and Futures Prices on:** | **22-Nov-93** |
| Futures Price: | 99.86 |
| Delivery Date: | 10-Dec-93 |
| Days: | 18 |
| **Cash Market:** | |
| Issue: | Bund |
| Maturity: | 15-Sep-2003 |
| Coupon: | 6 |
| Market Price: | 101.152 |
| Conversion Factor: | 0.999674 |
| Face Value: | DM250,000 |

First we must buy the bond in the cash market:

| | |
|---|---|
| Cash Market Price: | 101.152 |
| Times Face Value: | DM250,000 |
| Buy the Bund today and pay: | (DM252,880) |
| Plus Accrued Interest: | (DM2,792) |
| TOTAL Price | (DM255,672) |

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Next we sell the bonds to the exchange and calculate the proceeds we will receive on the delivery date. First we have to calculate the accrued interest:

| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 1) Choose the financial menu | | [FIN] | SELECT A MENU |
| 2) Choose the bond menu | | [BOND] | A/A SEMIANNUAL |
| 3) Set the type of bond to | | [TYPE] | |
| 30/360 annual | | [360] | |
| | | [ANN] | 30/360 ANNUAL |

| | | | |
|---|---|---|---|
| 4) Exit back to the bond menu | | [EXIT] | 30/360 ANNUAL |
| 5) Enter the delivery date for the December 1993 futures contract | 12.101993 | [SETT] | SETT=12.10.1993 FRI |
| 6) Enter the actual maturity date | 9.152003 | [MAT] | MAT=09/15/2003 MON |
| 7) Enter the coupon | 6 | [CPN%] | CPN%=6.000000 |
| 8) Change menus | | [MORE] | |
| 9) Solve for the accrued interest | | [ACCRU] | ACCRU=1.416667 |
| 10) Divide by | | [÷] | 1.416667÷ |
| 11) 100 times | 100 | [×] | 0.014167× |
| 12) The face value | 250000 | [=] | 3,541.666667 |

With accrued interest of DM3,542 the full delivery proceeds come to:

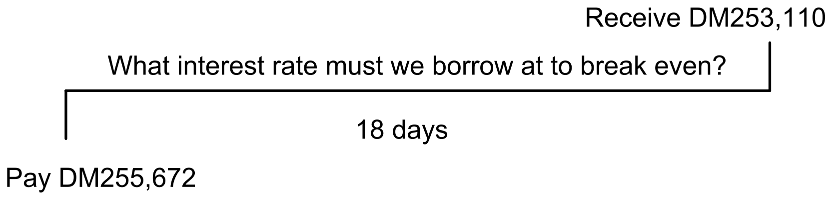
| | ***Value*** | ***Key*** | ***Display*** |
|---|---|---|---|
| 1) Multiply the current futures price times | 99.86 | [×] | 99.860000× |
| :2) The conversion factor divided by | 0.999674 | ÷ | 99.827446÷ |
| 3) 100 times | 100 | × | 0.998274× |
| 4) The face value plus | 250000 | + | 249,568.614100+ |
| 5) Accrued interest | 3541.666667 | = | 253,110.280767 |

The total delivery proceeds to the seller are DM253,110.28.

Did we make money? It depends on the financing cost. We can calculate the break-even financing cost as the interest rate which makes the money we must repay on the

delivery date equal to the proceeds we will receive from the futures exchange for selling the bonds.

This can be seen on a time line as follows:




We can calculate the rate using the calculator as follows:

| | | Value | Key | Display |
|---|---|---|---|---|
| 1) | Choose the financial menu | | [FIN] | SELECT A MENU |
| 2) | Choose the time value of money menu | | [TVM] | 1 PMTS/YR: END MODE |
| 3) | Enter the number of periods | 1 | [N] | N=1.0000 |
| 4) | Enter the present value as a negative number | 255672 | [+/–] [PV] | PV=–255,672.0000 |
| 5) | Enter the payments | 0 | [PMT] | PMT=0.0000 |
| 6) | Enter the future value | 253110 | [FV] | FV=53,110.0000 |
| 7) | Calculate the interest rate | | [I%YR] | I%YR=–1.0021 |
| 8) | Multiply by 360 | 360 | [×] | –360.7435 |
| 9) | Divide by the actual days | 18 | [÷] | –20.0413 |

This is the break-even financing rate: –20.04%.



If we can borrow money at any rate lower than the break-even rate, we can make money by buying the bond in the cash market and selling it in the futures market. With the repo rate at a level of 6.50%, this arbitrage would clearly not be profitable.

In order to make money by arbitraging this bond, we would have to be able to borrow money at a negative rate of interest, i.e. the lender would have to pay us to induce us to borrow the money.

2. Please calculate the implied repo rate, net basis and gross basis for the same bond.

**Implied Repo**

First we must buy the bond in the cash market:

| | |
|---|---|
| Cash Market Price: | 132 29/32 |
| Times Face Value: | $100,000.00 |
| Buy the Bund today and pay: | ($132,906.25) |
| Plus Accrued Interest: | ($25.21) |
| TOTAL Price | ($132,931.46) |

To calculate the accrued interest using the HP12C, you can do the following:

| | | *Value* | *Key* | *Display* |
|---|---|---|---|---|
| 1) | Clear financial registers and set accuracy to 4 digits | | [f][REG]<br>[f][4] | 0.0000 |
| 2) | Enter the first day of the current coupon period | 11.151993 | [ENTER] | 11.1520 |
| 3) | Enter the settlement date and calculate the number of days | 11.161993 | [g][ΔDYS] | 1.0000 |
| 4) | Store for future use | | [STORE][1] | 1.0000 |
| 5) | Enter the first day of the current coupon period | 11.151993 | [ENTER] | 11.1520 |
| 6) | Enter the last day of the current coupon period and calculate the number of days | 5.151994 | [g][ΔDYS] | 181.000 |
| 7) | Divide into 1 | | [1/*x*] | 0.0055 |
| 8) | Multiply by the number of days stored above | | [RCL][1][×] | 0.0055 |
| 9) | Multiply by the coupon | .09125 | [×] | 0.0005 |

| | | | |
|---|---|---|---|
| 10) Divide by 2 for the semi-annual coupon | 2 | [÷] | 0.0003 |
| 11) Multiply by the bond's face value | 100000 | [×] | 25.2072 |

The accrued interest amount is $25.21, as stated above.

Next we must sell the bond to the exchange and calculate the proceeds we will receive on the delivery date:

| | |
|---|---|
| Current Futures Price: | 115 19/32 |
| Times Conversion Factor: | 129.3725 |
| Times Face Value: | $129,372.53 |
| Plus Accrued Interest: | $3,428.18 |
| TOTAL Price | $132,800.70 |

The accrued interest of $3,428.18 is calculated as above, using a settlement date of 31 March 1994 instead of 16 November 1993.

Now we can calculate the implied repo rate for the 135-day period from 16 November 1993 to 31 March 1994:

| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 3) Enter the number of periods | 1 | [n] | 1.0000 |
| 4) Enter the present value as a negative number | 132931.46 | [CHS][PV] | –132,931.4600 |
| 5) Enter the payments | 0 | [PMT] | 0.0000 |
| 6) Enter the future value | 132800.7 | [FV] | 132,800.7000 |
| 7) Calculate the interest rate | | [i] | –0.0984 |
| 8) Multiply by 360 | 360 | [×] | –35.4119 |
| 9) Divide by the actual days | 135 | [÷] | –0.2623 |

The implied repo rate for the 9 1/8% Treasury due 15 May 2018 is –0.26%.

## Net Basis

To calculate net basis we must compare the all-in price of the bond in the cash market, financed at the market repo rate of 3.40%, to the all-in price of the bond in the futures market.

We already know the all-in price of the bond in the cash market: $132,931.46.

We can calculate the FV of this price at a rate of 3.40% for 135 days, and then calculate the basis expressed in price points of 1/32% by dividing it by the tick value of $31.25:

| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 1) Enter the number of periods | 1 | [n] | 1.0000 |
| 2) Enter the present value as a negative number | 132931.46 | [CHS][PV] | –132,931.4600 |
| 3) Enter the payments | 0 | [PMT] | 0.0000 |
| 4) Input the interest rate | 3.4 | [ENTER] | 3.4000 |
| 5) Multiply by the actual days | 135 | [×] | 459.0000 |
| 6) Divide by 360 and enter | 360 | [÷][i] | 1.2750 |
| 7) Calculate the future value | | [FV] | 134,626.3361 |
| 8) Change the sign | | [CHS] | –158,623.1942 |
| 9) Subtract from the all-in futures proceeds to get the net basis in $ | 132800.7 | [+] | –1,825.6361 |
| 10) Divide by the tick value of $31.25 to get the number of ticks | 31.25 | [÷] | –58.4204 |



The net basis is –58.42 ticks, which means the price in the cash market financed at the repo rate is 58 basis points higher than the price in the futures market.

$$\begin{array}{rr} +\text{Futures Price} & +\$129{,}372.53 \\ +\text{Accrued Interest} & +\$3{,}428.18 \\ -\text{Cash Price} & -\$132{,}906.25 \\ -\text{Accrued Interest} & -\$25.21 \\ -\text{Repo Interest} & -\$1{,}694.88 \\ \hline \text{Net Basis} & = -\$1{,}825.63 \end{array}$$

$$\text{Net Basis} = \frac{-1825.63}{31.25} = -58.42 \text{ ticks}$$

**<u>Gross Basis</u>**

<u>Comparing the gross price in the cash market to the gross price in the futures market gives us the *gross basis*:</u>

$$\begin{array}{rr} +\text{Futures Price} & +\$129{,}372.53 \\ -\text{Cash Price} & -\$132{,}906.25 \\ \hline \text{Gross Basis} & = -\$3{,}533.72 \end{array}$$

$$\text{Gross Basis} = \frac{-3{,}533.72}{31.25} = -113.08 \text{ ticks}$$

3. Please calculate how many March 1994 Treasury bond futures contracts you need to sell to hedge all price change in the following portfolio of Treasury bonds:

| Bond | Coupon | Maturity | Price | Yield | Modified Duration | Face Value | Market Value |
|---|---|---|---|---|---|---|---|
| Treasury | 6 1/4 | 15-Aug-2023 | 101 5/32 | 6.1638% | 13.3184 | $75,000,000 | $77,051,800.27 |
| Treasury | 8 1/2 | 15-Feb-2020 | 125 21/32 | 6.4569% | 11.7578 | $55,000,000 | $70,292,391.30 |
| Treasury | 12 1/2 | 15-Aug-2009 | 164 3/32 | 6.1050% | 8.5638 | $25,000,000 | $41,813,179.35 |
| Portfolio | | | | | 11.6875 | $155,000,000 | $189,157,370.92 |
| | | | | | | | |
| CTD | 11 3/4 | 15-Nov-2009 | 156 19/32 | 6.1436% | 8.8916 | | |

To hedge all price change in the above portfolio, we need to offset the entire duration of the portfolio. We can calculate the number of futures contracts we need to sell by using the modified duration hedge ratio formula:

$$\#C = -\frac{PV_U}{PV_{CTD}} \times \frac{MD_U}{MD_{CTD}} \times \frac{CF}{FV_F} \times \frac{\Delta i_U}{\Delta i_{CTD}}$$

Plugging in the numbers to the above equation yields the number of contracts we have to sell. In this case the underlying position is the entire portfolio, and we can use the market value and modified duration of the portfolio in the above formula.

But first we have to calculate the dirty price of the CTD. We will use the HP12C:

| | | ***Value*** | ***Key*** | ***Display*** |
|---|---|---|---|---|
| 1) | Clear financial registers and set accuracy to 4 digits | | [f][REG] [f][4] | 0.0000 |
| 2) | Enter the YTM | 6.1436 | [i] | 6.1436 |
| 3) | Enter the coupon | 11.75 | [PMT] | 11.7500 |
| 4) | Enter the settlement date | 11.161993 | [ENTER] | 11.1620 |
| 5) | Enter the maturity date and calculate the bond's price | 11.152009 | [f][PRICE] | 156.5931 |

6) Add the accrued interest [+] 156.6255

$$\#C = -\frac{\$189,157,370.92}{156.6255\%} \times \frac{11.6875}{8.8916} \times \frac{1.3298}{100,000} \times \frac{0.01\%}{0.01\%}$$

$$\#C = -2,111.00$$

4. How many contracts must you sell if you wish to decrease the duration of the portfolio owned to 5 years?

We can use the Hedge% formula to answer this question. We set the target duration to 5 and solve for the Hedge%:

$$\text{Hedge\%} = \frac{MD_U - MD_{Target}}{MD_U}$$

$$\text{Hedge\%} = \frac{11.6875 - 5}{11.6875} = 57.2193\%$$

We calculated above that we have to sell 2,111 March Treasury bond futures contracts to offset 100% of our underlying portfolio. We can therefore calculate how many contracts to sell in order to offset 57.2193%:

$$57.2193\% \times 2{,}111 = 1{,}207.90$$

We can therefore sell 1,208 contracts to reduce the duration of our portfolio to 5.

5. Please calculate the conversion factor for the following bond deliverable into the March 1994 Treasury bond futures contract:

| **Cash Market** | |
|---|---|
| Issue: | Treasury |
| Maturity: | 15-May-2018 |
| Settlement: | 16-Nov-1993 |
| Coupon: | 9 1/8 |
| Market Price: | 132 29/32 |
| Face Value: | $100,000.00 |

| **Futures Market** | |
|---|---|
| March 1993 Bund Futures Contract Price: | 115 19/32 |
| Settlement | 16-Nov-1993 |
| First Delivery Date: | 1-Mar-1994 |
| Days: | 135 |

The most important thing to remember is to adjust the maturity of the bond by bringing it back from the actual maturity of 15 May 2018 to the nearest even calendar quarter end. For this bond, that means calculating with an adjusted maturity date of 1 March 2018:

## Using the HP12C Calculator:

| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 1) Clear financial registers and set accuracy to 4 digits | | [f][REG] [f][4] | 0.0000 |
| 2) Enter 8% YTM | 8 | [i] | 8.0000 |
| 3) Enter the coupon | 9.125 | [PMT] | 11.7500 |
| 4) Enter the delivery date for the March 1994 futures contract | 3.011994 | [ENTER] | 3.0120 |
| 5) Enter the adjusted maturity date and calculate the bond's price | 3.012018 | [f][PRICE] | 111.9223 |

| | | | |
|---|---|---|---|
| 6) Divide by 100 | 100 | [÷] | 1.1192 |

This is the conversion factor.

## Using the HP19B Calculator:

| | *Value* | *Key* | *Display* |
|---|---|---|---|
| 1) Choose the financial menu | | [FIN] | SELECT A MENU |
| 2) Choose the bond menu | | [BOND] | A/A SEMIANNUAL |
| 3) Set the type of bond to | | [TYPE] | |
| actual/actual semi-annual | | [A/A] | |
| | | [SEMI] | A/A SEMIANNUAL |
| 4) Exit back to the bond menu | | [EXIT] | A/A SEMIANNUAL |
| 5) Enter the delivery date for the March 1994 futures contract | 3.011994 | [SETT] | SETT=03.01.1994 TUE |
| 6) Enter the adjusted maturity date | 3.012018 | [MAT] | MAT=03.01.2018 THU |
| 7) Enter the coupon | 9.125 | [CPN%] | CPN%=11.7500 |
| 8) Change menus | | [MORE] | |
| 9) Enter 8% YTM | 8 | [YLD%] | YLD%=8.0000 |
| 10) Solve for the price | | [PRICE] | PRICE=111.9223 |
| 11) Divide by 100 | 100 | [÷] | 1.1192 |